

Dual-Timescale Spectrum Management in Wireless Heterogeneous Networks

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April 10, 2016

Abstract

To attain the targeted data rates of next generation (5G) cellular networks requires dense deployment of small cells in addition to macro cells which provide wide coverage. Dynamic radio resource management (RRM) is crucial to the success of such heterogeneous networks due to much more pronounced traffic and interference variations in small cells. This work proposes a framework for spectrum management organized according to two timescales, which includes 1) centralized optimization on a second/minute timescale corresponding to typical durations of user sessions, and 2) distributed spectrum allocation on a millisecond timescale corresponding to typical latency requirements. The dual-timescale allocation is formulated as an optimization problem in the slower timescale with consideration of (distributed) opportunistic scheduling on the faster timescale. We consider both fixed and fully flexible user association. Iterative algorithms are developed to solve these optimization problems efficiently for a small cluster of cells. Simulation results demonstrate advantages of the dual-timescale RRM framework.

I. INTRODUCTION

Dense deployment of small cells is a promising means to address the scarcity of spectrum resources for wireless heterogeneous networks [1]. By reducing the coverage of each access point (AP) and increasing the density of APs, spectrum is reused more aggressively so that more links can transmit data simultaneously at higher rates, leading to much higher area spectral efficiency (bits/second/m²).

In existing 4G networks, radio resources allocation is usually based either on full spectrum

reuse or fractional frequency reuse (FFR). In FFR, a main portion of the spectrum is reused everywhere except at cell edge, and the remaining spectrum is divided for orthogonal reuse at cell edge [2]. Existing RRM techniques that take advantages of user and interference averaging in large cells are not as efficient in small cells. This is because traffic and interference variabilities are generally much more pronounced in smaller cells.

To address the highly dynamic traffic in small cells, we believe it is best to organize resource allocation according to traffic, channel, and interference conditions on two different timescales. On the slower timescale, a central controller allocates resources across many (e.g., hundreds of) cells periodically according to the average traffic distribution and channel conditions. On a faster timescale, each cell further schedule links within the cell onto the cell's allocated resources from the slow timescale. Due to latency requirements (often in milliseconds), the fast timescale allocation is most likely carried out in a distributed manner based on instantaneous local traffic and channel conditions within each small neighborhood of a few APs. Ideally, such updates should be sufficiently frequent to track the aggregate traffic volume of user sessions in each small area, yet also slow enough to allow collection of expected traffic and channel conditions across a large region and subsequent large-scale optimization. This timescale is conceived to be in seconds or a few minutes.

We introduce resource allocation for both fixed and flexible user association scenarios. Under the assumption of no fast timescale information exchange between cells, reference [3] formulated a slow timescale traffic-driven spectrum allocation problem, where full spectrum reuse and FFR are special cases. The treatment was generalized in [4] to jointly optimize user association and cell activation. Given the traffic intensities of all user equipments (UEs) that need to be supported, the controller basically seek the optimal set of *transmit patterns*. The pattern associated with a certain resource is defined as the subset of transmitters sharing it (they are granted access to the resource). Under mild assumptions, the optimization problem is shown to be convex and admits an extremely sparse solution, namely, very few patterns are activated and most UEs are associated to a single AP. References [3] and [4] assume the slow-timescale allocation is entirely in the frequency domain so that it does not interact with the fast-timescale allocation.

The goal of the current work is to study the interaction between the two timescales. In

particular, we allow instantaneous fast timescale information exchange between neighboring APs. This enables opportunistic scheduling, where an AP may use time resources not allocated to it when no neighbor it communicates with is busy. The formulation here accounts for interactions between the queues of different UEs. Since there is no known expression for the delay of such interactive queueing systems, an approximation is obtained by using the AP utilizations as a surrogate of the amount of interactions between queues. We gradually introduce three formulations with increasing capabilities and complexities. The overall optimization problem is essentially bi-convex, so we propose iterative algorithms to solve it with relatively low computational complexity and guaranteed convergence. Simulation results demonstrate substantial delay reduction compared to the schemes in [3].

This work is quite different from existing work on resource allocation in the literature. In particular, most authors, such as those of [5]–[8], consider spectrum allocation on a single timescale. The authors of [9] devise scheduling policies in the time domain, but consider full spectrum reuse only. The spectrum allocation problem is often formulated as a discrete optimization problem (in contrast to the continuous one here), e.g., [10]–[12]. The joint user association and spectrum allocation is explored because of their coupled nature [4], [13]–[17]. In particular, by assuming that all cells use all spectrum, [13] investigate a joint user association and intra-cell resource allocation problem. In [14], joint multi-cell channel allocation and user association is studied. However, their study is restricted to three pre-defined resource allocation strategies, namely, orthogonal deployment, co-channel deployment, and partially shared deployment. Although [4], [15], [17] consider resource allocation assigning arbitrary spectrum resources to links from an AP to its UEs, the treatments are for a single timescale.

The rest of this paper is organized as follows. The system model is introduced in Section II. Three increasing complex problem formulations are presented in Sections III, IV, and V, respectively. Numerical results are given in Section VI. Concluding remarks are given in Section VII.

II. SYSTEM MODEL

We consider the downlink of a wireless heterogeneous network with n access points, including possibly macro, pico, and other small cell base transceiver stations. Denote the set of all APs as $\mathcal{N} = \{1, \dots, n\}$. Two APs are neighbors of each other if they can exchange their traffic and

channel state information with negligible latency. Following [3], [4], we treat user equipments near each other with similar quality of service (QoS) requirements as a UE group. We then model the aggregate traffic of each UE group using a single queue on a slow timescale. We denote the set of all UE groups as $\mathcal{K} = \{1, \dots, k\}$. The traffic of UE group j is modeled by Poisson traffic arrivals with arrival rate λ^j . The packet length is exponentially distributed with average length L .

Suppose APs operate on a (licensed) frequency band of W Hz. The frequency resources are assumed to be homogeneous on the slow timescale. Any spectrum allocation to the n APs can be viewed as a division of the spectrum into 2^n segments corresponding to all possible patterns. Let $y_{\mathcal{F}}$ be the fraction of total bandwidth allocated to pattern \mathcal{F} . We have $\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1$, and any efficient allocation should set the bandwidth of the empty pattern to zero, i.e., $y_{\emptyset} = 0$.

As in [4], each AP may serve any subset of UE groups and each UE group may be served by any subset of APs. Throughout this paper, we assume APs do not support broadcast coding schemes, that is, if an AP transmits data to multiple UE groups simultaneously, those transmissions must be over nonoverlapping spectrum. As a result, over any slice of spectrum, it may use it to transmit to one of the k UE groups or none of them. If a chunk of spectrum is shared by a set of APs, denoted as \mathcal{F} , then every AP in \mathcal{F} may divide the spectrum into up to k orthogonal segments and assign them to different UE groups. Denote the bandwidth used by AP i to serve UE group j over frequency pattern \mathcal{F} as $x_{\mathcal{F}}^{i \rightarrow j}$. Then, for every $\mathcal{F} \subset \mathcal{N}$ and $i \in \mathcal{F}$,

$$\sum_{j \in \mathcal{K}} x_{\mathcal{F}}^{i \rightarrow j} = y_{\mathcal{F}}. \quad (1)$$

An example of spectrum allocation for three APs is shown Fig. 1, where $y_{\{1,3\}}$ is the fraction of total bandwidth allocated to pattern $\{1, 3\}$. AP 1 assigns $x_{\{1,3\}}^{1 \rightarrow 1}$ to UE group 1 and the rest under pattern $\{1, 3\}$ to UE group 2, while AP 3 assigns $x_{\{1,3\}}^{3 \rightarrow 1}$ to UE group 1 and the rest under pattern $\{1, 3\}$ to UE group 2.

An AP is said to be *busy* (resp. *idle*) at a time if it have (resp. has no) data to transmit to its associated UEs. An AP is said to be *active* over a certain resource if the AP actually occupies the resource to transmit. Let $s_{\mathcal{A}}^{i \rightarrow j}$ denote the spectral efficiency of the link from AP i to UE group j over pattern \mathcal{A} (i.e., when all APs in \mathcal{A} are active over the spectrum). Assume that AP i transmits

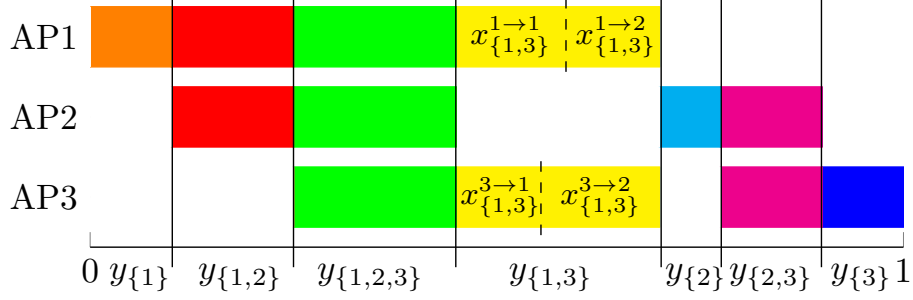


Fig. 1: Spectrum allocation for three APs.

with fixed flat transmit power spectral density (PSD) q^i . It is also assumed that if multiple APs transmit to the same UE over the same spectrum, then data from each AP is decoded with other transmissions treated as interferences. For concreteness in obtaining numerical results, we use Shannon's formula to obtain the spectral efficiency of the link from AP i to UE group j over pattern \mathcal{A} :

$$s_{\mathcal{A}}^{i \rightarrow j} = \frac{W \mathbf{1}(i \in \mathcal{A})}{L} \log_2 \left(1 + \frac{q^i h^{i \rightarrow j}}{\sum_{i' \in \mathcal{A} \setminus \{i\}} q^{i'} h^{i' \rightarrow j} + n^j} \right) \quad (2)$$

in packets per second where $\mathbf{1}(i \in \mathcal{A}) = 1$ if $i \in \mathcal{A}$ and $\mathbf{1}(i \in \mathcal{A}) = 0$ otherwise, $h^{i \rightarrow j}$ is the power gain of the link from AP i to UE group j , and n^j is the noise PSD at UE group j . On the slow timescale considered in this paper, the link gain $h^{i \rightarrow j}$ is treated as constant and flat over the entire spectrum, which includes the effects of path loss and shadowing. If AP i serves a single fixed UE group, we drop the UE group index j and use $s_{\mathcal{A}}^i$ to represent the spectral efficiency of the link under active AP set \mathcal{A} . It should become clear that we distinguish resources based on the patterns because the actual set of APs transmitting over a resource determine the quality of all links over the pattern.

A unique feature in this paper is to consider allocation of time resources to patterns in a similar fashion as spectrum allocation. Let $z_{\mathcal{T}}$ be the fraction of time allocated to time pattern \mathcal{T} . Clearly, $\sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} = 1$. Time allocation is different from spectrum allocation, since time scheduling induces extra delay. Here we assume the granularity of a time unit is small in comparison to packet transmission time, thus we can ignore scheduling delay.

In general, the time allocation can be different for different spectrum segments. To prevent

the treatment from becoming unwieldy, we assume identical time allocation for all spectrum segments. As a result, we simply divide all resources into orthogonal rectangular time-frequency physical resource blocks (PRBs) in similar manner as in LTE. Each PRB is indexed by $(\mathcal{F}, \mathcal{T})$, where \mathcal{F} is the pattern in frequency and \mathcal{T} the pattern in time. The bandwidth of the PRB is $W \cdot y_{\mathcal{F}}$ Hz and the duration is $z_{\mathcal{T}}$ time units.

If resource allocation occurs only on the slow timescale, it is apparently inconsequential whether the allocation is in time, in frequency, or in both. However, the actual allocation can be further adjusted on a fast timescale depending on instantaneous traffic. We consider a simple and fair scheduling rule: AP i transmits on a PRB indexed by $(\mathcal{F}, \mathcal{T})$ only if the following conditions hold:

- The spectrum block of this PRB is assigned to AP i , i.e., $i \in \mathcal{F}$;
- AP i is busy (it has data to transmit);
- Either the time block of this PRB is assigned to AP i , i.e., $i \in \mathcal{T}$, or all neighbors of AP i do not data to transmit on this PRB.

This rule allows the APs to opportunistically employ more resources when their neighbors are silent.

An example allocation to two neighboring APs is shown in Fig. 2. The spectrum segments $\{1\}$ and $\{2\}$ are used exclusively. The spectrum segment $\{1, 2\}$ is shared between the two APs, which is further divided into three time patterns: $\{1\}$, $\{2\}$, and $\{1, 2\}$. What is interesting is that the time resources can be reallocated opportunistically depending on instantaneous traffic conditions. Specifically, The time-frequency resource allocation at time unit t_1 corresponds to the case where both APs are busy, and that at time unit t_2 corresponds to the case where only AP 2 is busy, where AP 2 takes the spectrum shared by the two AP's, including the time patterns allocated to AP 1.

III. SLOW-TIMESCALE SPECTRUM ALLOCATION WITH FIXED USER ASSOCIATION

We start with slow-timescale spectrum allocation with fixed user association, which is the simplest model in this paper. For simplicity, we assume that each AP serves one UE group that has been associated to it. An AP is silent when it has no traffic to its assigned UE groups. At any point in time, every AP adapts its rate to the instantaneous set of active APs \mathcal{A} . The rate

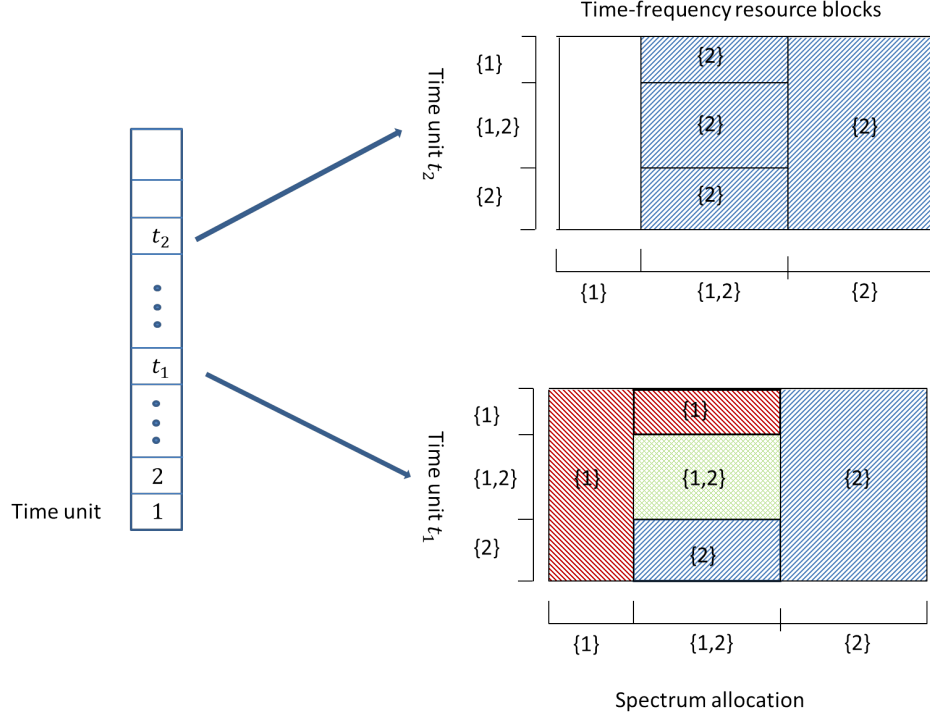


Fig. 2: Time-frequency resource blocks for two neighboring APs: the set in the shadowed area represents the set of active APs in each resource block.

contributed by frequency pattern \mathcal{F} to AP i is $s_{\mathcal{F} \cap \mathcal{A}}^i y_{\mathcal{F}}$, i.e., the efficiency multiplied by the pattern's bandwidth. Hence, the total service rate of AP i under active AP set \mathcal{A} is

$$r_{\mathcal{A}}^i = \sum_{\mathcal{F} \subset \mathcal{N}} s_{\mathcal{F} \cap \mathcal{A}}^i y_{\mathcal{F}} \text{ packets/second.} \quad (3)$$

Each AP has a queue of packets to transmit with arrival rate λ^i .

The analysis of interactive queues is an open problem. There is no known explicit expression of average delay in general interactive-queue model. To make progress, we assume that the events that different APs transmit are independent. In a stable interactive queuing system, AP i transmits over a fraction of time, which is referred to as the utilization of AP i and devoted as ρ_i . The probability that the set of busy APs is \mathcal{A} is approximated as

$$p_{\mathcal{A}} = \left(\prod_{i \in \mathcal{A}} \rho_i \right) \prod_{i' \notin \mathcal{A}} (1 - \rho_{i'}). \quad (4)$$

It is useful to note that

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} p_{\mathcal{A}} = \rho_i. \quad (5)$$

Thus, $p_{\mathcal{A}}/\rho_i$ is the fraction of time that the pattern is \mathcal{A} when $i \in \mathcal{A}$ is active.

We develop an approximation for the average packet delay in the interactive queueing system. When AP i transmits packets, its service rate is chosen from 2^{n-1} possible values which correspond to different sets of busy APs. Assuming that the service rate is constant during transmission of each packet, we use a M/G/1 queue with 2^{n-1} classes of packets to approximate the average delay in interactive queues. Each class of packets corresponds to a specific set of busy APs and service rates. The arrival rate of packets associated with pattern \mathcal{A} (which includes i) is estimated as $\frac{p_{\mathcal{A}}}{\rho_i} \lambda^i$. From [18], the utilization of the M/G/1 queue is

$$\rho_i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i}. \quad (6)$$

Evidently, a stable queue must satisfy $\rho_i < 1$.

Lemma 1. *The average delay of the M/G/1 queue is*

$$d^i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right). \quad (7)$$

Lemma 1 is proved in Appendix A. It is important to note that the right hand side of (7) is convex in $(r_{\mathcal{A}}^i)_{\mathcal{A} \subset \mathcal{N}}$. Hereafter we let the objective of resource allocation be the minimization of the approximate average delay (7).

Problem 1 (Spectrum allocation over a slow timescale).

$$\underset{\mathbf{y}, \mathbf{r}, \boldsymbol{\rho}, \mathbf{p}, \mathbf{d}}{\text{minimize}} \sum_{i=1}^n \lambda^i d^i \quad (8a)$$

$$\text{subject to } d^i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right), \quad \forall i \in \mathcal{N} \quad (8b)$$

$$r_{\mathcal{A}}^i = \sum_{\mathcal{F} \subset \mathcal{N}} s_{\mathcal{F} \cap \mathcal{A}}^i y_{\mathcal{F}}, \quad \forall i \in \mathcal{N}, \quad \forall \mathcal{A} \subset \mathcal{N} \quad (8c)$$

$$y_{\mathcal{F}} \geq 0, \quad \forall \mathcal{F} \subset \mathcal{N} \quad (8d)$$

$$\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1, \quad (8e)$$

$$p_{\mathcal{A}} = \left(\prod_{i \in \mathcal{A}} \rho_i \right) \prod_{j \notin \mathcal{A}} (1 - \rho_j), \quad \forall \mathcal{A} \subset \mathcal{N} \quad (8f)$$

$$\rho_i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i}, \quad \forall i \in \mathcal{N}. \quad (8g)$$

The variables in Problem 1 are $\mathbf{y} = (y_{\mathcal{F}})_{\mathcal{F} \subset \mathcal{N}}$, $\mathbf{r} = (r_{\mathcal{A}}^i)_{\mathcal{A} \subset \mathcal{N}, i \in \mathcal{N}}$, $\boldsymbol{\rho} = (\rho_i)_{i \in \mathcal{N}}$, $\mathbf{p} = (p_{\mathcal{A}})_{\mathcal{A} \subset \mathcal{N}}$, and $\mathbf{d} = (d^i)_{i \in \mathcal{N}}$. The objective (8a), if normalized by the total arrival rate $\sum_{i=1}^n \lambda^i$, is the (approximate) average packet delay of the entire network.

Lemma 2. *The solution to Problem 1 remains the same if constraint (8g) is relaxed as*

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i} \leq \rho_i, \quad \forall i \in \mathcal{N}. \quad (9)$$

The proof of Lemma (2) is relegated to Appendix B.

Problem 1 may be nonconvex due to the nonlinear-equality constraints. However, we can use an iterative method to solve Problem 1 with low complexity. We divide Problem 1 into two subproblems. Each subproblem corresponds to one group of variables. The basic idea of the proposed algorithm is to alternatively update the two groups of variables as described in below.

Subproblem 1 (Fix ρ and \mathbf{p} , update \mathbf{y} and \mathbf{r}).

$$\underset{\mathbf{y}, \mathbf{r}}{\text{minimize}} \sum_{\mathcal{A} \subset \mathcal{N}} \sum_{i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right) \quad (10a)$$

$$\text{subject to } r_{\mathcal{A}}^i = \sum_{\mathcal{F} \subset \mathcal{N}} s_{\mathcal{F} \cap \mathcal{A}}^i y_{\mathcal{F}}, \quad \forall i \in \mathcal{N}, \quad \forall \mathcal{A} \subset \mathcal{N} \quad (10b)$$

$$y_{\mathcal{F}} \geq 0, \quad \forall \mathcal{F} \subset \mathcal{N} \quad (10c)$$

$$\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1, \quad (10d)$$

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i} \leq \rho_i, \quad \forall i \in \mathcal{N}. \quad (10e)$$

The variables in Subproblem 1 are \mathbf{y} and \mathbf{r} . Subproblem 1 is a convex optimization problem because the terms on the left hand side of (10e) is convex, all other constraints are linear and each objective is a linear combination of convex functions. Thus, each subproblem has a unique global minimum when feasible [19].

Given \mathbf{y} and \mathbf{r} , we update ρ and \mathbf{p} through an iterative algorithm. Here we use the method of interference function in [20] to update ρ and \mathbf{p} . Define $\mathbf{f}(\rho)$ as a vector of n functions, and the i th function as

$$f^i(\rho) = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \left[\left(\prod_{l \in \mathcal{A}, l \neq i} \rho_l \right) \prod_{i' \notin \mathcal{A}} (1 - \rho_{i'}) \right] \frac{\lambda^i}{r_{\mathcal{A}}^i}. \quad (11)$$

We obtain the similar properties of function $\mathbf{f}(\cdot)$ as the interference function in [20].

Lemma 3. *If $1 \geq \rho, \hat{\rho} \geq 0$ and $\rho \geq \hat{\rho}$, then $\mathbf{f}(\rho) \geq \mathbf{f}(\hat{\rho})$.*

Lemma 3 is proved in Appendix C. Next, we update ρ and \mathbf{p} by using the following algorithm.

Algorithm 1 (Fix \mathbf{y} and \mathbf{r} , update ρ and \mathbf{p}). :

1. *Initialization: set $\rho^{(0)}$ as current AP utilizations;*
2. *Update utilization and probabilities:*

$$\rho^{(m+1)} = \mathbf{f}(\rho^{(m)}) \quad (12)$$

3. If $|\boldsymbol{\rho}^{(m)} - \boldsymbol{\rho}^{(m-1)}| \geq \epsilon$, where ϵ is a small number, go back to step 2; otherwise stop and let

$$\rho_i = \rho_i^{(m)}, \forall i \in \mathcal{N} \quad (13)$$

$$p_{\mathcal{A}} = \left(\prod_{l \in \mathcal{A}} \rho_l^{(m)} \right) \prod_{i' \notin \mathcal{A}} (1 - \rho_{i'}^{(m)}), \forall \mathcal{A} \subset \mathcal{N}. \quad (14)$$

Lemma 4. *Algorithm 1 converges when it starts with $\boldsymbol{\rho}^{(0)}$ satisfying $\boldsymbol{\rho}^{(0)} \geq \mathbf{f}(\boldsymbol{\rho}^{(0)})$.*

Lemma 4 is proved in Appendix D.

Equipped with the preceding solution for the subproblems, the proposed iterative algorithm for solving Problem 1 is described as follows:

Algorithm 2 (Iterative algorithm for solving Problem 1). :

1. *Initialization:* $\rho_i = 1, \forall i \in \mathcal{N}$; $p_{\mathcal{A}} = 1$ if $\mathcal{A} = \mathcal{N}$, $p_{\mathcal{A}} = 0$ otherwise;
2. *Update \mathbf{y} and \mathbf{r} by solving Subproblem 1;*
3. *Update $\boldsymbol{\rho}$ and \mathbf{p} by using Algorithm 1;*
4. *Terminate if \mathbf{y} has converged. Otherwise, return to step 2.*

Algorithm 2 mainly deals with the non-convexity nature of Problem 1. Algorithm 1 converges relatively fast due to the properties of $\mathbf{f}(\cdot)$ in Lemma 1. Thus, Problem 1 can be solved in an iterative way with manageable complexity as long as n is not large.

Theorem 1. *Algorithm 2 converges to a fixed point.*

Theorem 1 is proved in Appendix E. The solution derived through Algorithm 2 may or may not be a local or global minimum. The effectiveness of the proposed framework and solution is validated using simulation results in Section VI.

IV. DUAL-TIMESCALE ALLOCATION WITH FIXED USER ASSOCIATION

In this section, we extend the model in Section III to describe a dual-timescale resource allocation problem. While the resource allocation optimization is still carried out periodically on the same slow timescale as before, the effect of opportunistic scheduling on the fast timescale is addressed. It is assumed that, given the spectrum and time resources allocated on the slow timescale, each cell exchanges instantaneous information about queue length and channel state

with its neighboring cells, and adjusts resource allocation accordingly on a fast timescale.

As illustrated in Fig. 2, time and frequency resources can be regarded as a collection of 2-dimensional blocks. Every AP adapts its rate to the set of busy APs, denoted as \mathcal{A} . $\mathcal{F} \cap \mathcal{A}$ is the set of APs allowed to transmit under pattern \mathcal{F} in frequency. On PRB $(\mathcal{F}, \mathcal{T})$, all APs in $\mathcal{F} \cap \mathcal{T}$ with data will transmit, whereas those without data remain silent. If some APs scheduled for PRB $(\mathcal{F}, \mathcal{T})$ have no data to send, their selected neighbors take their places to transmit. This replacement selection can be itself a fast timescale optimization problem. For concreteness, we introduce a specific simple selection method. As described in Section II, if AP i is busy and is allowed to use frequency pattern \mathcal{F} (i.e., $i \in \mathcal{F}$), and all its neighboring APs have no data to send on PRB $(\mathcal{F}, \mathcal{T})$, then AP i uses PRB $(\mathcal{F}, \mathcal{T})$ to transmit. According to this rule, the set of active APs on PRB $(\mathcal{F}, \mathcal{T})$ is determined by $(\mathcal{F}, \mathcal{T}, \mathcal{A})$.

In this section, we continue to assume that only one UE group is associated to each AP. Denote $\eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i$ as the spectral efficiency of AP i on PRB $(\mathcal{F}, \mathcal{T})$ when the set of busy APs is \mathcal{A} . In this case, suppose the actual set of active APs on PRB $(\mathcal{F}, \mathcal{T})$ as \mathcal{B} according to the replacement selection method, then we have

$$\eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i = s_{\mathcal{B}}^i. \quad (15)$$

The average service rate over one time unit is considered for analyzing the average delay. The rate of AP i over PRB $(\mathcal{F}, \mathcal{T})$ is $\eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i y_{\mathcal{F}} z_{\mathcal{T}}$. Hence, the average service rate of AP i when the set of busy APs is \mathcal{A} is calculated as

$$r_{\mathcal{A}}^i = \sum_{\mathcal{F} \subset \mathcal{N}, \mathcal{T} \subset \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i y_{\mathcal{F}} z_{\mathcal{T}} \text{ packets/second.} \quad (16)$$

Problem 2 (Spectrum and time allocation with fixed user association).

$$\underset{\mathbf{y}, \mathbf{z}, \mathbf{r}, \boldsymbol{\rho}, \mathbf{p}, \mathbf{d}}{\text{minimize}} \sum_{i=1}^n \lambda^i d^i \quad (17a)$$

$$\text{subject to } d^i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right), \quad \forall i \in \mathcal{N} \quad (17b)$$

$$r_{\mathcal{A}}^i = \sum_{\mathcal{F} \subset \mathcal{N}, \mathcal{T} \subset \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i y_{\mathcal{F}} z_{\mathcal{T}}, \quad \forall i \in \mathcal{N}, \quad \forall \mathcal{A} \subset \mathcal{N} \quad (17c)$$

$$\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1, \quad (17d)$$

$$\sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} = 1, \quad (17e)$$

$$z_{\mathcal{T}} \geq 0, \quad \forall \mathcal{T} \subset \mathcal{N} \quad (17f)$$

$$y_{\mathcal{F}} \geq 0, \quad \forall \mathcal{F} \subset \mathcal{N} \quad (17g)$$

$$p_{\mathcal{A}} = \left(\prod_{i \in \mathcal{A}} \rho_i \right) \prod_{i' \notin \mathcal{A}} (1 - \rho_{i'}), \quad \forall \mathcal{A} \subset \mathcal{N} \quad (17h)$$

$$\rho_i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \left[\left(\prod_{l \in \mathcal{A}, l \neq i} \rho_l \right) \prod_{i' \notin \mathcal{A}} (1 - \rho_{i'}) \right] \frac{\lambda^i}{r_{\mathcal{A}}^i}, \quad \forall i \in \mathcal{N}. \quad (17i)$$

The variables in Problem 2 are \mathbf{y} , \mathbf{z} , \mathbf{r} , $\boldsymbol{\rho}$, \mathbf{p} and \mathbf{d} . In comparison with Problem 1, here we have additional variables \mathbf{z} and corresponding constraints (17e) and (17f). Because of time allocation, $r_{\mathcal{A}}^i$ depends on both \mathbf{y} and \mathbf{z} . Problem 2 is in general nonconvex due to the nonlinear-equality constraints. Similar to Section III, we can use an iterative method to solve Problem 2. We divide Problem 2 into three subproblems. Each subproblem corresponds to one group of variables.

Subproblem 2.1 (Fix ρ , \mathbf{p} and \mathbf{z} , update \mathbf{y} , \mathbf{r}).

$$\underset{\mathbf{y}, \mathbf{r}}{\text{minimize}} \quad \sum_{\mathcal{A} \subset \mathcal{N}} \sum_{i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right) \quad (18a)$$

$$\text{subject to } r_{\mathcal{A}}^i = \sum_{\mathcal{F} \subset \mathcal{N}} \left(\sum_{\mathcal{T} \subset \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i z_{\mathcal{T}} \right) y_{\mathcal{F}}, \quad \forall i \in \mathcal{N}, \quad \forall \mathcal{A} \subset \mathcal{N} \quad (18b)$$

$$y_{\mathcal{F}} \geq 0, \quad \forall \mathcal{F} \subset \mathcal{N} \quad (18c)$$

$$\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1, \quad (18d)$$

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i} \leq \rho_i, \quad \forall i \in \mathcal{N}. \quad (18e)$$

Subproblem 2.1 is identical to Subproblem 1 with $s_{\mathcal{F} \cap \mathcal{A}}^i$ replaced by $\eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i$.

Subproblem 2.2 (Fix ρ , \mathbf{p} and \mathbf{y} , update \mathbf{z} , \mathbf{r}).

$$\underset{\mathbf{z}, \mathbf{r}}{\text{minimize}} \quad \sum_{\mathcal{A} \subset \mathcal{N}} \sum_{i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right) \quad (19a)$$

$$\text{subject to } r_{\mathcal{A}}^i = \sum_{\mathcal{T} \subset \mathcal{N}} \left(z_{\mathcal{T}} \sum_{\mathcal{F} \subset \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{A}}^i y_{\mathcal{F}} \right), \quad \forall i \in \mathcal{N}, \quad \forall \mathcal{A} \subset \mathcal{N} \quad (19b)$$

$$z_{\mathcal{T}} \geq 0, \quad \forall \mathcal{T} \subset \mathcal{N} \quad (19c)$$

$$\sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} = 1, \quad (19d)$$

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i} \leq \rho_i, \quad \forall i \in \mathcal{N}. \quad (19e)$$

The variables in Subproblem 2.2 are \mathbf{z} and \mathbf{r} .

Subproblems 2.1 and 2.2 are convex optimization problems. Thus, each subproblem has a unique global minimum when feasible. Given \mathbf{y} , \mathbf{z} and \mathbf{r} , we update ρ and \mathbf{p} using Algorithm 1. After introducing each update in the proposed method, we illustrate an iterative algorithm to solve Problem 2.2:

Algorithm 3 (Iterative algorithm for solving Problem 2). :

1. *Initialization:* $\rho_i = 1, \forall i \in \mathcal{N}$; $p_{\mathcal{A}} = 1$ if $\mathcal{A} = \mathcal{N}$, $p_{\mathcal{A}} = 0$ otherwise;
2. *Update \mathbf{y} , \mathbf{r} by solving Subproblem 2.1;*
3. *Update \mathbf{z} , \mathbf{r} by solving Subproblem 2.2;*

4. Update ρ, \mathbf{p} by using Algorithm 1;
5. Terminate if \mathbf{y} has converged. Otherwise, return to step 2.

Subproblems 2.1 and 2.2 have 2^n variables after eliminating intermediate variables \mathbf{r} . Similar to Problem 1, Problem 2 can be solved in an iterative way with manageable complexity.

Theorem 2. *Algorithm 3 converges to a fixed point.*

Theorem 2 is proved in Appendix F. Similarly, the solution derived through Algorithm 3 may or may not be a local or global minimum.

V. JOINT USER ASSOCIATION AND DUAL-TIMESCALE ALLOCATION

In this section, we complete the model for dual-timescale resource allocation with fully flexible user association. Each AP can serve multiple UE groups at the same time, and each UE group can be served simultaneously by multiple APs. We say an AP is an *interfering AP* at a UE group if the AP is busy and interferes at the UE group over all or parts of the spectrum.

We analyze average packet delay from the perspective of an individual UE group. We let the service rate of the UE group depend only on the set of interfering APs. We consider UE group j . Assume the set of interfering APs at UE group j is \mathcal{I} , and AP i is transmitting data to UE group j under frequency pattern \mathcal{F} . Then, the set of active APs under frequency pattern \mathcal{F} is calculated as $\mathcal{A} = (\mathcal{F} \cap \mathcal{I}) \cup \{i\}$. On the time-frequency slot $(\mathcal{F}, \mathcal{T})$, all APs in $\mathcal{A} \cap \mathcal{T}$ transmits data. Some APs scheduled for the time-frequency slot $(\mathcal{F}, \mathcal{T})$ may have no data to send, so their selected neighbors are allowed to take their places to transmit. We use the same rule for selecting replacements as in Section IV. Denote the set of APs active on the time-frequency slot $(\mathcal{F}, \mathcal{T})$ under the set of interfering APs \mathcal{I} be \mathcal{C} .

The spectral efficiency on time-frequency slot $(\mathcal{F}, \mathcal{T})$ from AP i to UE group j under the set of interfering APs \mathcal{I} is denoted as

$$\eta_{\mathcal{F}, \mathcal{T}, \mathcal{I}}^{i \rightarrow j} = s_C^{i \rightarrow j}. \quad (20)$$

The service rate of UE group j contributed by AP i on the time-frequency slot $(\mathcal{F}, \mathcal{T})$ is $\eta_{\mathcal{F}, \mathcal{T}, \mathcal{I}}^{i \rightarrow j} z_{\mathcal{T}} x_{\mathcal{F}}^{i \rightarrow j}$. Hence, the total service rate of UE group j given the set of interfering APs \mathcal{I} is

calculated as

$$r_{\mathcal{I}}^j = \sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} \sum_{\mathcal{F} \subset \mathcal{N}} \left(\sum_{i \in \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{I}}^{i \rightarrow j} x_{\mathcal{F}}^{i \rightarrow j} \right), \quad \forall j \in \mathcal{K}, \quad \forall \mathcal{I} \subset \mathcal{N}. \quad (21)$$

We still assume that the events that different APs transmit are independent and approximate the probability of interfering APs \mathcal{I} as

$$p_{\mathcal{I}} = \left(\prod_{i \in \mathcal{I}} \rho_i \right) \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}). \quad (22)$$

All traffic for UE group j arrives with a rate of λ^j at a queue. Queues are interactive among UE groups. In a stable interactive queuing system, each UE group receives packets for a portion of time, and keeps silent for the rest. We denote the utilization of UE group j as σ_j . If the system is stable, σ_j is smaller than one. Here we take advantage of $(\sigma_j)_{j \in \mathcal{K}}$ to improve the average delay. Since the service rate of each UE group depends on the set of interfering APs, service rate of UE group j is chosen from at most 2^n possible values corresponding to different sets of interfering APs. Thus, we use a M/G/1 queue with 2^n classes of packets to approximate the average delay of UE group j in interactive queues, similar to Sections III and IV. Each class corresponds to a specific set of interfering APs to UE group j . Different classes are associated with different service rates and arrival rates. According to formula (11) in [18], the average delay of UE group j using the M/G/1-queue approximation is

$$\hat{d}^j = \sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}} \left(\left(\frac{1}{r_{\mathcal{I}}^j} \right)^2 \frac{\lambda^j}{1 - \sigma_j} + \frac{1}{r_{\mathcal{I}}^j} \right). \quad (23)$$

In addition, the utilization of UE group j is approximated as

$$\sigma_j = \sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}} \frac{\lambda^j}{r_{\mathcal{I}}^j}. \quad (24)$$

If AP i serves UE group j under reuse pattern \mathcal{F} , then the utilization of AP i on spectrum segment $x_{\mathcal{F}}^{i \rightarrow j}$ is σ_j . Because AP i may serve multiple UE group, its utilization may be different on different parts of spectrum. The average utilization of AP i under frequency pattern \mathcal{F} is estimated as $\frac{1}{y_{\mathcal{F}}} \sum_{j \in \mathcal{K}} \sigma_j x_{\mathcal{F}}^{i \rightarrow j}$. For simplicity, we estimate the overall utilization of AP i as the

average utilization over the spectrum assigned to AP i

$$\rho_i = \frac{1}{\sum_{\mathcal{F}: i \in \mathcal{F}} y_{\mathcal{F}}} \sum_{\mathcal{F}: i \in \mathcal{F}} \sum_{j \in \mathcal{K}} \sigma_j x_{\mathcal{F}}^{i \rightarrow j} \quad (25)$$

where $\sum_{\mathcal{F}: i \in \mathcal{F}} y_{\mathcal{F}}$ is the total bandwidth allocated to AP i .

With the preceding approximations, the joint dual-timescale resource allocation and user association problem is formulated as:

Problem 3 (Joint dual-timescale allocation and user association).

$$\underset{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{r}, \boldsymbol{\sigma}, \mathbf{p}, \hat{\mathbf{d}}}{\text{minimize}} \quad \sum_{j=1}^k \lambda^j \hat{d}^j \quad (26a)$$

$$\text{subject to } \hat{d}^j = \sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}} \left(\left(\frac{1}{r_{\mathcal{I}}^j} \right)^2 \frac{\lambda^j}{1 - \sigma_j} + \frac{1}{r_{\mathcal{I}}^j} \right), \quad \forall j \in \mathcal{K} \quad (26b)$$

$$r_{\mathcal{I}}^j = \sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} \sum_{\mathcal{F} \subset \mathcal{N}} \left(\sum_{i \in \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{I}}^{i \rightarrow j} x_{\mathcal{F}}^{i \rightarrow j} \right), \quad \forall j \in \mathcal{K}, \quad \forall \mathcal{I} \subset \mathcal{N} \quad (26c)$$

$$\sum_{j \in \mathcal{K}} x_{\mathcal{F}}^{i \rightarrow j} = y_{\mathcal{F}}, \quad \forall i \in \mathcal{N}, \mathcal{F} \subset \mathcal{N} \quad (26d)$$

$$\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1, \quad (26e)$$

$$\sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} = 1, \quad (26f)$$

$$z_{\mathcal{T}} \geq 0, \quad \forall \mathcal{T} \subset \mathcal{N} \quad (26g)$$

$$y_{\mathcal{F}}, x_{\mathcal{F}}^{i \rightarrow j} \geq 0, \quad \forall i \in \mathcal{N}, j \in \mathcal{K}, \mathcal{F} \subset \mathcal{N} \quad (26h)$$

$$p_{\mathcal{I}} = \left(\prod_{i \in \mathcal{I}} \rho_i \right) \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}), \quad \forall \mathcal{I} \subset \mathcal{N} \quad (26i)$$

$$\rho_i = \frac{1}{\sum_{\mathcal{F}: i \in \mathcal{F}} y_{\mathcal{F}}} \sum_{\mathcal{F}: i \in \mathcal{F}} \sum_{j \in \mathcal{K}} \sigma_j x_{\mathcal{F}}^{i \rightarrow j}, \quad \forall i \in \mathcal{N} \quad (26j)$$

$$\sigma_j = \sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}} \frac{\lambda^j}{r_{\mathcal{I}}^j}, \quad \forall j \in \mathcal{K}. \quad (26k)$$

The variables in Problem 3 are $\mathbf{x} = (x_{\mathcal{F}}^{i \rightarrow j})_{i \in \mathcal{N}, j \in \mathcal{K}, \mathcal{F} \subset \mathcal{N}}$, $\mathbf{y}, \mathbf{z}, \mathbf{r} = (r_{\mathcal{I}}^j)_{\mathcal{I} \subset \mathcal{N}, j \in \mathcal{K}}$, $\boldsymbol{\sigma} = (\sigma_j)_{j \in \mathcal{K}}$, $\boldsymbol{\rho}, \mathbf{p} = (p_{\mathcal{I}})_{\mathcal{I} \subset \mathcal{N}}$ and $\hat{\mathbf{d}} = (\hat{d}^j)_{j \in \mathcal{K}}$. In comparison with Problem 2, here we have new variables \mathbf{x} and $\boldsymbol{\sigma}$, and new constraints (26d), (26j) and (26k). Because each UE group may be served by

multiple APs, here $r_{\mathcal{I}}^j$ is contributed by several APs. Problem 3 may be nonconvex due to the nonlinear-equality constraints. Similar to in the previous sections, we can use iterative method to solve the problem with low complexity. We divide Problem 3 into three subproblems. Each subproblem corresponds to one group of variables.

Subproblem 3.1 (Fix σ , ρ , \mathbf{p} and \mathbf{z} , update \mathbf{x} , \mathbf{y} and \mathbf{r}).

$$\underset{\mathbf{x}, \mathbf{y}, \mathbf{r}}{\text{minimize}} \quad \sum_{\mathcal{I} \subset \mathcal{N}} \sum_{j \in \mathcal{K}} p_{\mathcal{I}} \lambda^j \left(\left(\frac{1}{r_{\mathcal{I}}^j} \right)^2 \frac{\lambda^j}{1 - \sigma_j} + \frac{1}{r_{\mathcal{I}}^j} \right) \quad (27a)$$

$$\text{subject to } r_{\mathcal{I}}^j = \sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} \sum_{\mathcal{F} \subset \mathcal{N}} \left(\sum_{i \in \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{I}}^{i \rightarrow j} x_{\mathcal{F}}^{i \rightarrow j} \right), \quad \forall j \in \mathcal{K}, \quad \forall \mathcal{I} \subset \mathcal{N} \quad (27b)$$

$$\sum_{j \in \mathcal{K}} x_{\mathcal{F}}^{i \rightarrow j} = y_{\mathcal{F}}, \quad \forall i \in \mathcal{N}, \mathcal{F} \subset \mathcal{N} \quad (27c)$$

$$\sum_{\mathcal{F} \subset \mathcal{N}} y_{\mathcal{F}} = 1, \quad (27d)$$

$$y_{\mathcal{F}}, x_{\mathcal{F}}^{i \rightarrow j} \geq 0, \quad \forall i \in \mathcal{N}, j \in \mathcal{K}, \mathcal{F} \subset \mathcal{N} \quad (27e)$$

$$\sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}} \frac{\lambda^j}{r_{\mathcal{I}}^j} \leq \sigma_j, \quad \forall j \in \mathcal{K}. \quad (27f)$$

The variables in Subproblem 3.2 are \mathbf{x} , \mathbf{y} and \mathbf{r} .

Subproblem 3.2 (Fix σ , ρ , \mathbf{p} , \mathbf{x} and \mathbf{y} , update \mathbf{z} and \mathbf{r}).

$$\underset{\mathbf{z}, \mathbf{r}}{\text{minimize}} \quad \sum_{\mathcal{I} \subset \mathcal{N}} \sum_{j \in \mathcal{K}} p_{\mathcal{I}} \lambda^j \left(\left(\frac{1}{r_{\mathcal{I}}^j} \right)^2 \frac{\lambda^j}{1 - \sigma_j} + \frac{1}{r_{\mathcal{I}}^j} \right) \quad (28a)$$

$$\text{subject to } r_{\mathcal{I}}^j = \sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} \sum_{\mathcal{F} \subset \mathcal{N}} \left(\sum_{i \in \mathcal{N}} \eta_{\mathcal{F}, \mathcal{T}, \mathcal{I}}^{i \rightarrow j} x_{\mathcal{F}}^{i \rightarrow j} \right), \quad \forall j \in \mathcal{K}, \quad \forall \mathcal{I} \subset \mathcal{N} \quad (28b)$$

$$\sum_{\mathcal{T} \subset \mathcal{N}} z_{\mathcal{T}} = 1, \quad (28c)$$

$$z_{\mathcal{T}} \geq 0, \quad \forall \mathcal{T} \subset \mathcal{N} \quad (28d)$$

$$\sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}} \frac{\lambda^j}{r_{\mathcal{I}}^j} \leq \sigma_j, \quad \forall j \in \mathcal{K}. \quad (28e)$$

The variables in Subproblem 3.2 are \mathbf{z} and \mathbf{r} .

Subproblems 3.1 and 3.2 are convex optimization problems. Given \mathbf{x} , \mathbf{y} , \mathbf{z} and \mathbf{r} , we update σ , ρ and \mathbf{p} through an iterative method similar to Algorithm 1:

Algorithm 4 (Fix $\mathbf{x}, \mathbf{y}, \mathbf{z}$ and \mathbf{r} , update $\boldsymbol{\sigma}$, $\boldsymbol{\rho}$ and \mathbf{p}). :

1. *Initialization: set $\boldsymbol{\sigma}^{(0)}$ as the current UE group utilizations;*
2. *Update utilization and probabilities:*

$$\sigma_j^{(m+1)} = \sum_{\mathcal{I} \subset \mathcal{N}} p_{\mathcal{I}}^{(m)} \frac{\lambda_j^{\mathcal{I}}}{r_{\mathcal{I}}^j}, \quad \forall j \in \mathcal{K}, \quad (29)$$

$$\rho_i^{(m+1)} = \frac{1}{\sum_{\mathcal{F}: i \in \mathcal{F}} y_{\mathcal{F}}} \sum_{\mathcal{F}: i \in \mathcal{F}} \sum_{j \in \mathcal{K}} \sigma_j^{(m+1)} x_{\mathcal{F}}^{i \rightarrow j}, \quad \forall i \in \mathcal{N} \quad (30)$$

$$p_{\mathcal{I}}^{(m+1)} = \prod_{i \in \mathcal{I}} \rho_i^{(m+1)} \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}^{(m+1)}), \quad \forall \mathcal{I} \subset \mathcal{N}. \quad (31)$$

3. *If $|\boldsymbol{\sigma}^{(m)} - \boldsymbol{\sigma}^{(m-1)}| \geq \epsilon$, where ϵ is a small number, go back to step 2; otherwise stop and obtain*

$$\sigma_j = \sigma_j^{(m)}, \quad \rho_i = \rho_i^{(m)}, \quad p_{\mathcal{I}} = p_{\mathcal{I}}^{(m)}, \quad \forall j \in \mathcal{K}, i \in \mathcal{N}, \mathcal{I} \subset \mathcal{N}. \quad (32)$$

According to (29), (30) and (31), we can construct a function $\mathbf{g}(\cdot)$ such that

$$\boldsymbol{\sigma}^{(m+1)} = \mathbf{g}(\boldsymbol{\sigma}^{(m)}). \quad (33)$$

We first explore the properties of function $\mathbf{g}(\cdot)$ as Lemma 3.

Lemma 5. *If $1 \geq \boldsymbol{\sigma}, \hat{\boldsymbol{\sigma}} \geq 0$ and $\boldsymbol{\sigma} \geq \hat{\boldsymbol{\sigma}}$, then $\mathbf{g}(\boldsymbol{\sigma}) \geq \mathbf{g}(\hat{\boldsymbol{\sigma}})$.*

Lemma 5 is proved in Appendix C. Then, we prove the convergence of Algorithm 4.

Lemma 6. *Algorithm 4 converges when it starts with $\boldsymbol{\sigma}^{(0)}$ satisfying $\boldsymbol{\sigma}^{(0)} \geq \mathbf{g}(\boldsymbol{\sigma}^{(0)})$.*

Lemma 6 is proved in Appendix G

After introducing each subproblem in the proposed method, we illustrate an iterative algorithm to solve Problem 3:

Algorithm 5 (Iterative algorithm for solving Problem 3). :

1. *Initialization: set $\sigma_j = 1, \forall j \in \mathcal{K}; \rho_i = 1, \forall i \in \mathcal{N}; p_{\mathcal{I}} = 1$ if $\mathcal{I} = \mathcal{N}$, $p_{\mathcal{I}} = 0$ otherwise;*
2. *Update \mathbf{x}, \mathbf{y} and \mathbf{r} by solving Subproblem 3.1;*
3. *Update \mathbf{z} and \mathbf{r} by solving Subproblem 3.2;*
4. *Update $\boldsymbol{\sigma}, \boldsymbol{\rho}$ and \mathbf{p} using Algorithm 4;*

5. *Terminate if \mathbf{x} has converged. Otherwise, return to step 2.*

Subproblems 3.1 and 3.2 are convex optimization problems. Also, Algorithm 4 converges relatively fast due to Lemma 5. Thus, Problem 3 can be solved in an iterative way with manageable complexity.

Theorem 3. *Algorithm 5 converges to a fixed point.*

Theorem 3 is proved in Appendix H.

In this section, we have considered a formulation with reduced number of service rates, and designed a practical algorithm with lower complexity in compare with the complete model for dual-timescale resource allocation. Since there are n APs and each AP has $k + 1$ choices for a small slice of the spectrum, the system has $(k + 1)^n$ usage choices on the slice of spectrum. To precisely calculate the service rates, the bandwidth for $(k + 1)^n$ usage choices needs to be determined. A switch between on/off status of one UE group changes its associated APs' spectrum usage. As a result, the interference and services rates at other UE groups may be affected. Thus, the service rate of a particular UE group depends on the set of UE groups having non-empty queues. If we consider all possible service rates, the computational complexity may be very high due to a large number of variables and states.

VI. SIMULATION RESULTS

A system with 8 APs is depicted in Fig. 5. As introduced in Section II, an AP may transmit over time resources allocated to others as long as owners of the resource have no data to transmit.

First we show the performance of the proposed resource allocation for fixed user association. A special network is studied, where each AP is associated with one randomly placed UE group. The proposed resource allocations (slow-timescale allocations, and dual-timescale resource allocation) are compared with three other schemes: 1) full spectrum reuse (as in LTE), 2) the conservative allocation in [3], and 3) the refined allocation in [3]. In the experiments, interactive queues are simulated where service rates adapt to instantaneous interference. Fig. 4 depicts average packet delay against different traffic loads for all four different allocations. As expected, the proposed dual-timescale allocation (with opportunistic fast timescale scheduling) has the best performance under all traffic conditions. The refined allocation and slow-timescale allocation

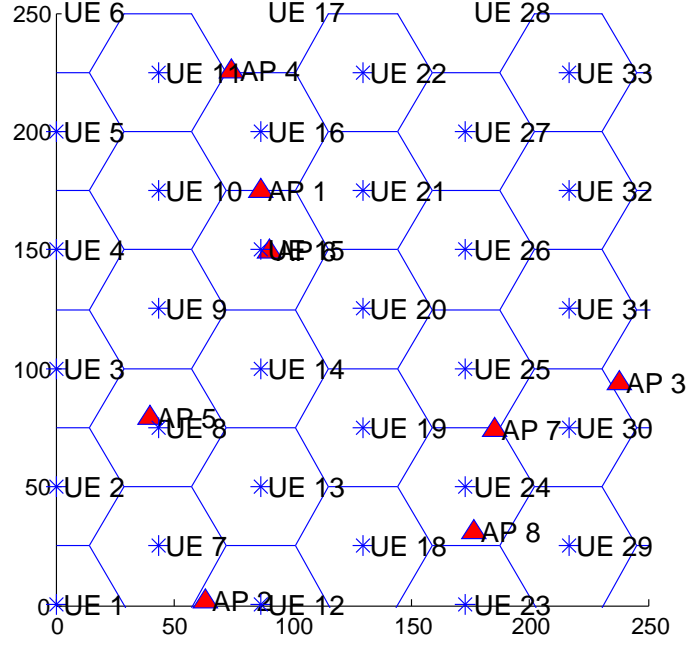


Fig. 3: A system with 8 APs and 33 UE groups.

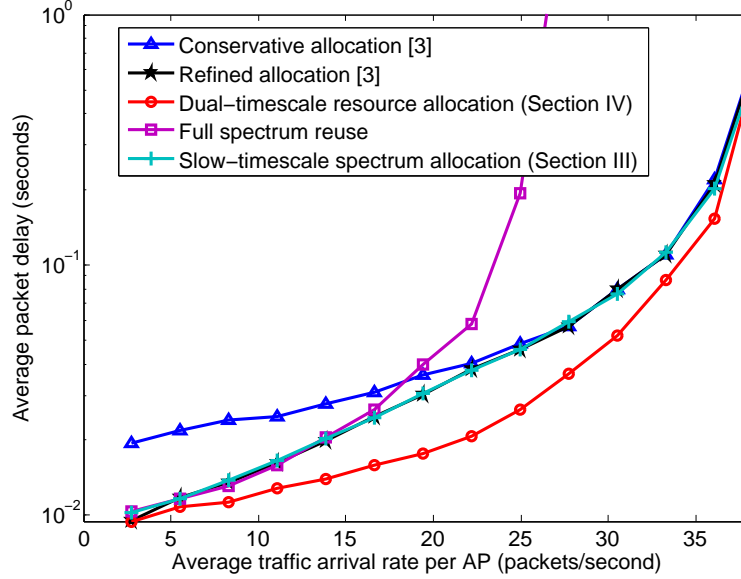


Fig. 4: Delay performance of different allocation methods with fixed user association as a function of average network traffic load.

using the M/G/1-queue approximation are the close second in both cases of very light traffic and very heavy traffic, but leads to much higher delay in case of moderate traffic. Indeed, this case is when it is the most beneficial to consider the impact of fast timescale scheduling when making slow timescale allocations.

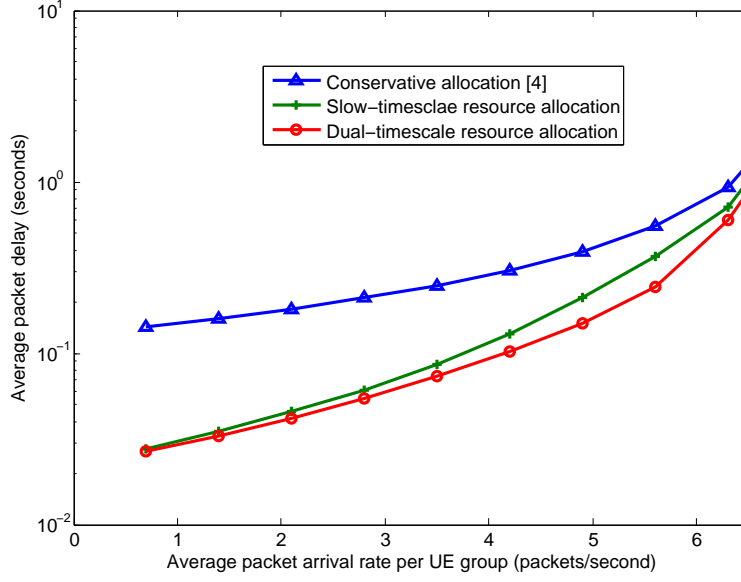


Fig. 5: Delay performance of joint resource allocation and user association.

Next, we show the performance of the joint resource allocation and user association. We consider a system with 8 APs and 33 UE groups. Resource allocation by the proposed algorithm is compared with the conservative model in [4]. Interactive queues are simulated by using the solution to the proposed optimization problems. Simulation results are illustrated in Fig. 5. Resource allocation over a slow timescale, which is a special case of the formulation in Section V with $z_N = 1$, has marginal gain over the conservative model. With dual-timescale allocation, the average delay is further reduced. As the average packet arrival rate increases, the gain by dual-timescale allocation goes up first, then reduces. When the average arrival rate per UE group is 5.5 packets per second, we observe about 30% gain for dual-timescale resource allocation over slow-timescale resource allocation. Based on observations in the experiments, the gain by dual-timescale allocation is substantial in the interference-limited case. It is because that with dual-timescale time allocation, APs can take advantage of opportunistic scheduling on a fast timescale and achieve much higher rate when their neighbors have no data to send.

VII. CONCLUSION

We have reported some new modeling, problem formulation, and techniques for traffic-driven radio resource management in wireless heterogeneous networks. At the core is a global slow timescale optimization with considerations of effects of dynamic scheduling on the fast timescale

as well as utilization of allocated resources. Exploiting these factors leads to significant quality of service improvements. The improvements are obtained at the cost of increased computational complexity. The scheme is readily implemented in a small network. Extension of the work to larger networks is left to future work.

APPENDIX A

PROOF OF LEMMA 1

We prove this lemma using the delay analysis for M/G/c queue with multiple customer classes in [18]. According to formula (11) in [18], the approximation of waiting time for general case is

$$W = \frac{\left(\frac{EV^2}{2EV}\right) c^{-1} B}{1 - c^{-1} \rho}, \quad (34)$$

where ρ is the utilization, V is the service time random variable, B is the probability of delay in the M/M/c system with the same expected service time. The approximation formula (34) is exact in the M/G/1 case [18].

According to our formulation, AP i has 2^{n-1} classes of packages. The package with service rate $r_{\mathcal{A}}^i$ corresponds to the arrival rate $\frac{p_{\mathcal{A}} \lambda^i}{\rho_i}$. Therefore, the utilization equals to

$$\rho_i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i}. \quad (35)$$

The expected service time is

$$EV = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i r_{\mathcal{A}}^i}, \quad (36)$$

and the second moment of service time is

$$EV^2 = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{2p_{\mathcal{A}}}{\rho_i (r_{\mathcal{A}}^i)^2}. \quad (37)$$

Since $c = 1$, the probability of delay in the M/M/1 queue with the same expected service time

(36) is

$$B = \lambda^i EV = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i}. \quad (38)$$

From (34), (35), (36), (37), and (38), the average delay of AP i under our formulation is

$$d^i = EV + W \quad (39)$$

$$= \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i r_{\mathcal{A}}^i} + \frac{\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{2p_{\mathcal{A}}}{\rho_i (r_{\mathcal{A}}^i)^2}}{2 \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i r_{\mathcal{A}}^i}} \cdot \frac{\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i}}{1 - \rho_i} \quad (40)$$

$$= \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right). \quad (41)$$

APPENDIX B

PROOF OF LEMMA 2

We prove the lemma by contradiction. Assume the solution to the relaxed optimization problem is $\mathbf{y}, \mathbf{r}, \boldsymbol{\rho}, \mathbf{p}, \mathbf{d}$. If the lemma is not true, then there exists i^* such that $\rho_{i^*} > \sum_{\mathcal{A} \subset \mathcal{N}: i^* \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^{i^*}}{\rho_{i^*} j \notin \mathcal{A}}$.

Define

$$\rho'_i = \begin{cases} \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i}, & \text{if } i = i^* \\ \rho_i, & \text{otherwise,} \end{cases} \quad (42)$$

and

$$p'_{\mathcal{A}} = \prod_{i \in \mathcal{A}} \rho'_i \prod_{j \notin \mathcal{A}} (1 - \rho'_j). \quad (43)$$

Assuming $i^* \in \mathcal{B}$, we have $p'_B + p'_{\mathcal{B} \setminus \{i^*\}} = p_B + p_{\mathcal{B} \setminus \{i^*\}}$, $p'_B < p_B$ and $p'_{\mathcal{B} \setminus \{i^*\}} > p_{\mathcal{B} \setminus \{i^*\}}$. According to the fact that $r_{\mathcal{B}}^i \leq r_{\mathcal{B} \setminus \{i^*\}}^i$ if $i \in \mathcal{B}$ and $i \neq i^*$, we have

$$\sum_{\mathcal{A} \in \{\mathcal{B}, \mathcal{B} \setminus \{i^*\}\}} \frac{p_{\mathcal{A}} \lambda^i}{\rho_i r_{\mathcal{A}}^i} > \sum_{\mathcal{A} \in \{\mathcal{B}, \mathcal{B} \setminus \{i^*\}\}} \frac{p'_{\mathcal{A}} \lambda^i}{\rho'_i r_{\mathcal{A}}^i}, \quad (44)$$

$$\sum_{\mathcal{A} \in \{\mathcal{B}, \mathcal{B} \setminus \{i^*\}\}} \frac{p_{\mathcal{A}}}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right) > \sum_{\mathcal{A} \in \{\mathcal{B}, \mathcal{B} \setminus \{i^*\}\}} \frac{p'_{\mathcal{A}}}{\rho'_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho'_i} + \frac{1}{r_{\mathcal{A}}^i} \right). \quad (45)$$

Hence,

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \frac{\lambda^i}{r_{\mathcal{A}}^i} > \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p'_{\mathcal{A}}}{\rho'_i} \frac{\lambda^i}{r_{\mathcal{A}}^i}, \quad (46)$$

$$\sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho_i} + \frac{1}{r_{\mathcal{A}}^i} \right) > \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p'_{\mathcal{A}}}{\rho'_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho'_i} + \frac{1}{r_{\mathcal{A}}^i} \right). \quad (47)$$

Therefore, for any $i \neq i^*$,

$$\rho'_i = \rho_i \geq \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p_{\mathcal{A}}}{\rho_i} \frac{\lambda^i}{r_{\mathcal{A}}^i} > \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p'_{\mathcal{A}}}{\rho'_i} \frac{\lambda^i}{r_{\mathcal{A}}^i} \quad (48)$$

where the equality holds according to definition of ρ' ; the first inequality holds because $\mathbf{y}, \mathbf{r}, \boldsymbol{\rho}, \mathbf{p}, \mathbf{d}$ must satisfy the relaxed constraint; the second inequality holds according to (46). From (43), if $i^* \in \mathcal{A}$, then $\frac{p'_{\mathcal{A}}}{\rho'_{i^*}} = \frac{p_{\mathcal{A}}}{\rho_{i^*}}$. Thus, according to (42), $\rho'_{i^*} = \sum_{\mathcal{A} \subset \mathcal{N}: i^* \in \mathcal{A}} \frac{p'_{\mathcal{A}}}{\rho'_{i^*}} \frac{\lambda^{i^*}}{r_{\mathcal{A}}^{i^*}}$.

Define $\hat{d}^i = \sum_{\mathcal{A} \subset \mathcal{N}: i \in \mathcal{A}} \frac{p'_{\mathcal{A}}}{\rho'_i} \left(\left(\frac{1}{r_{\mathcal{A}}^i} \right)^2 \frac{\lambda^i}{1 - \rho'_i} + \frac{1}{r_{\mathcal{A}}^i} \right)$. From (42) and (43), $d'_{i^*} \leq d_{i^*}$. According to (47), we have $\hat{d}^i < d^i$, if $i \neq i^*$. The solution to $\mathbf{y}, \mathbf{r}, \boldsymbol{\rho}', \mathbf{p}', \mathbf{d}'$ is in the feasible region of the relaxed problem and results in a smaller objective value. Therefore, it contradicts with the assumption that $\mathbf{y}, \mathbf{r}, \boldsymbol{\rho}, \mathbf{p}, \mathbf{d}$ is the optimal solution. Hence, this proves the lemma.

APPENDIX C

PROOFS OF LEMMAS 3 AND 5

We first prove Lemma 5, which is for a more general case than Lemma 3.

If $\boldsymbol{\sigma} \geq \hat{\boldsymbol{\sigma}}$, then $\sigma_j \geq \hat{\sigma}_j$ for all $j \in \mathcal{K}$. Create $\boldsymbol{\sigma}'$ such that

$$\sigma'_j = \begin{cases} \hat{\sigma}_j, & \text{if } j = 1 \\ \sigma_j, & \text{otherwise.} \end{cases} \quad (49)$$

For any $i \in \mathcal{N}$, we denote

$$\rho'_i = \frac{1}{\sum_{\mathcal{F}: i \in \mathcal{F}} y_{\mathcal{F}}} \sum_{\mathcal{F}: i \in \mathcal{F}} \sum_{j \in \mathcal{K}} \sigma'_j x_{\mathcal{F}}^{i \rightarrow j}. \quad (50)$$

According to (25) and (50), since $\boldsymbol{\sigma}' \geq \boldsymbol{\sigma}$, then $\boldsymbol{\rho}' \geq \boldsymbol{\rho}$. Denote $g^j(\boldsymbol{\sigma})$ as the j th element of $\mathbf{g}(\boldsymbol{\sigma})$.

From (29), (30) and (31), we have

$$g^j(\boldsymbol{\sigma}) = \sum_{\mathcal{I} \subset \mathcal{N}} \left[\prod_{l \in \mathcal{I}} \rho_l \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}) \right] \frac{\lambda^j}{r_{\mathcal{I}}^j}. \quad (51)$$

We show $g^j(\boldsymbol{\sigma}') \geq g^j(\boldsymbol{\sigma})$ for every $j \in \mathcal{K}$. Construct $\boldsymbol{\rho}''$ such that

$$\rho''_i = \begin{cases} \rho'_i, & \text{if } i = 1 \\ \rho_i, & \text{otherwise.} \end{cases} \quad (52)$$

Assume that $1 \in \mathcal{I}_1 \subset \mathcal{K}$. Denote $\mathcal{I}_2 = \mathcal{I}_1 \setminus \{1\}$. According to the definition of $\boldsymbol{\rho}''$ in (50),

$$\prod_{l \in \mathcal{I}_1} \rho''_l \prod_{i' \notin \mathcal{I}_1} (1 - \rho''_{i'}) \geq \prod_{l \in \mathcal{I}_1} \rho_l \prod_{i' \notin \mathcal{I}_1} (1 - \rho_{i'}), \quad (53)$$

$$\prod_{l \in \mathcal{I}_2} \rho''_l \prod_{i' \notin \mathcal{I}_2} (1 - \rho''_{i'}) \leq \prod_{l \in \mathcal{I}_2} \rho_l \prod_{i' \notin \mathcal{I}_2} (1 - \rho_{i'}), \quad (54)$$

$$\sum_{\mathcal{I} \in \{\mathcal{I}_1, \mathcal{I}_2\}} \prod_{l \in \mathcal{I}} \rho''_l \prod_{i' \notin \mathcal{I}} (1 - \rho''_{i'}) = \sum_{\mathcal{I} \in \{\mathcal{I}_1, \mathcal{I}_2\}} \prod_{l \in \mathcal{I}} \rho_l \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}). \quad (55)$$

Also, since $\mathcal{I}_2 \subset \mathcal{I}_1$, $r_{\mathcal{I}_1}^j \leq r_{\mathcal{I}_2}^j$. Hence, we have

$$\sum_{\mathcal{I} \in \{\mathcal{I}_1, \mathcal{I}_2\}} \prod_{l \in \mathcal{I}} \rho''_l \prod_{i' \notin \mathcal{I}} (1 - \rho''_{i'}) \frac{\lambda^i}{r_{\mathcal{I}}^i} \leq \sum_{\mathcal{I} \in \{\mathcal{I}_1, \mathcal{I}_2\}} \prod_{l \in \mathcal{I}} \rho_l \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}) \frac{\lambda^i}{r_{\mathcal{I}}^i}. \quad (56)$$

Since \mathcal{K} can be grouped into pairs $\{\mathcal{I}_1, \mathcal{I}_2\}$,

$$\sum_{\mathcal{I} \subset \mathcal{K}} \prod_{l \in \mathcal{I}} \rho''_l \prod_{i' \notin \mathcal{I}} (1 - \rho''_{i'}) \frac{\lambda^i}{r_{\mathcal{I}}^i} \leq \sum_{\mathcal{I} \subset \mathcal{K}} \prod_{l \in \mathcal{I}} \rho_l \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}) \frac{\lambda^i}{r_{\mathcal{I}}^i}. \quad (57)$$

By induction, we can prove

$$\sum_{\mathcal{I} \subset \mathcal{K}} \prod_{l \in \mathcal{I}} \rho'_l \prod_{i' \notin \mathcal{I}} (1 - \rho'_{i'}) \frac{\lambda^i}{r_{\mathcal{I}}^i} \leq \sum_{\mathcal{I} \subset \mathcal{K}} \prod_{l \in \mathcal{I}} \rho_l \prod_{i' \notin \mathcal{I}} (1 - \rho_{i'}) \frac{\lambda^i}{r_{\mathcal{I}}^i}. \quad (58)$$

According to (51), it is equivalent to $g^j(\boldsymbol{\sigma}') \leq g^j(\boldsymbol{\sigma})$.

Thus, $\mathbf{g}(\boldsymbol{\sigma}') \leq \mathbf{g}(\boldsymbol{\sigma})$. By induction, we can prove $\mathbf{g}(\widehat{\boldsymbol{\sigma}}) \leq \mathbf{g}(\boldsymbol{\sigma})$. Hence, Lemma 5 is proved.

Lemma 3 is a special case of Lemma 5, in which each AP serves only one UE. In this case, $k = n$ and $x_{\mathcal{F}}^{i \rightarrow i} = y_{\mathcal{F}}$ for any $\mathcal{F} \subset \mathcal{N}$. Thus, we have $\boldsymbol{\sigma} = \boldsymbol{\rho}$ and $\mathbf{g}(\cdot) = \mathbf{f}(\cdot)$. Lemma 3 can be proved in the similar method.

APPENDIX D

PROOF OF LEMMA 4

From Algorithm 1, $\boldsymbol{\rho}^{(1)} = \mathbf{f}(\boldsymbol{\rho}^{(0)})$. If $\boldsymbol{\rho}^{(0)} \geq \mathbf{f}(\boldsymbol{\rho}^{(0)})$, then according to Lemma 3

$$\boldsymbol{\rho}^{(1)} = \mathbf{f}(\boldsymbol{\rho}^{(0)}) \geq \mathbf{f}(\mathbf{f}(\boldsymbol{\rho}^{(0)})) = \mathbf{f}(\boldsymbol{\rho}^{(1)}) = \boldsymbol{\rho}^{(2)}. \quad (59)$$

By induction, for any $m = 0, 1, 2, \dots$,

$$\boldsymbol{\rho}^{(m+1)} \geq \boldsymbol{\rho}^{(m)}. \quad (60)$$

Because series $(\boldsymbol{\rho}^{(m)})_{m \in \mathcal{N}}$ is lower-bounded by zero and non-increasing, the series must converge.

APPENDIX E

PROOF OF THEOREM 1

First, we prove Subproblem 1 in the step 2 of Algorithm 2 is always feasible. In the first iteration, we have $\boldsymbol{\rho} = \mathbf{1}$ in step 2. It is easy to check that Subproblem 1 in the first iteration is feasible.

We will show that if Subproblem 1 in the m th iteration is feasible, then it is also feasible in the $(m + 1)$ st iteration. Assume that we fix $\boldsymbol{\rho}$ and \mathbf{p} in step 2 of the m th iteration. Assuming Subproblem 1 in the m th iteration is feasible, the solution satisfies (10e). From (4), (10e) and (11), the updated \mathbf{y} and \mathbf{r} in step 2 result in $\mathbf{f}(\boldsymbol{\rho}) \leq \boldsymbol{\rho}$. Then, at the beginning of step 3 of the m th iteration, we set $\boldsymbol{\rho}^{(0)} = \boldsymbol{\rho}$. Since $\mathbf{f}(\boldsymbol{\rho}^{(0)}) \leq \boldsymbol{\rho}^{(0)}$, Algorithm 1 in the step 3 of the m th iteration converges according to Lemma 4 and the updated $\boldsymbol{\rho}$ is smaller than $\boldsymbol{\rho}^{(0)}$ and satisfy (4) and (6). The algorithm goes to step 2 of the $(m + 1)$ st iteration, with the updated $\boldsymbol{\rho}$ and \mathbf{p} in step 3 of the m th iteration, (10e) holds. Thus, the current \mathbf{y} and \mathbf{r} satisfy all the constraint in Subproblem 1 in step 2 of the $(m + 1)$ st iteration. Therefore, Subproblem 1 is always feasible.

Next, from the preceding analysis, both steps 2 and 3 result in non-increasing $\boldsymbol{\rho}$ which is lower-bounded by zero. Hence, Algorithm 2 converges to a fixed point.

APPENDIX F

PROOF OF THEOREM 2

The prove is similar to that for Theorem 1.

First, we prove Subproblem 2.1 and 2.2 in Algorithm 3 are always feasible. In the first iteration, we have $\rho = 1$ in step 2. It is easy to check that Subproblem 2.1 is feasible in the first iteration.

We will prove that if Subproblem 2.1 in the m th iteration is feasible, Subproblem 2.2 in the m th iteration and Subproblem 2.1 in the $(m+1)$ st iteration are feasible. Assume that we fix \mathbf{z} , ρ and \mathbf{p} in step 2 of the m th iteration. Assuming Subproblem 2.1 is feasible, the solution satisfies (18e). Fixing \mathbf{x} , ρ and \mathbf{p} , the current \mathbf{z} and \mathbf{r} satisfy all the constraints in Subproblem 2.2 in step 3 of the m th iteration. Therefore, Subproblem 2.2 of the m th iteration is feasible and the solution satisfies (19e). From (4), (11) and (19e), the updated \mathbf{y} in step 2, \mathbf{z} and \mathbf{r} in step 3 result in $\mathbf{f}(\rho) \leq \rho$. Then, at the beginning of step 4, we have $\rho^{(0)} = \rho$. Since $\mathbf{f}(\rho^{(0)}) \leq \rho^{(0)}$, Algorithm 3 in step 4 of the m th iteration converges according to Lemma 4 and the updated ρ is smaller than $\rho^{(0)}$ and satisfies (4) and (6). If the $(m+1)$ st iteration is conducted, the current \mathbf{y} , \mathbf{z} , \mathbf{r} , ρ and \mathbf{p} satisfy (18e) in Subproblem 2.1. Thus, it is straightforward that Subproblem 2.1 of the $(m+1)$ st iteration is feasible.

Next, from the preceding analysis, both steps 2, 3 and 4 result in non-increasing ρ which is lower-bounded by zero. Hence, Algorithm 3 converges.

APPENDIX G

PROOF OF LEMMA 6

From Algorithm 4, $\sigma^{(1)} = \mathbf{g}(\sigma^{(0)})$. If $\sigma^{(0)} \geq \mathbf{g}(\sigma^{(0)})$, then according to Lemma 5

$$\sigma^{(1)} = \mathbf{g}(\sigma^{(0)}) \geq \mathbf{g}(\mathbf{g}(\sigma^{(0)})) = \mathbf{g}(\sigma^{(1)}) = \sigma^{(2)}. \quad (61)$$

By induction, for any $m = 0, 1, 2, \dots$,

$$\sigma^{(m+1)} \geq \sigma^{(m)}. \quad (62)$$

Because series $(\sigma^{(m)})_{m \in \mathcal{N}}$ is lower-bounded by zero and non-increasing, the series must converge.

APPENDIX H

PROOF OF THEOREM 3

The prove is similar to that for Theorem 2.

First, we prove Subproblem 3.1 and 3.2 in Algorithm 5 are always feasible. In the first iteration, we have $\sigma = 1$ in step 2. It is easy to check that Subproblem 3.1 is feasible in the first iteration.

We will prove that if Subproblem 3.1 in the m th iteration is feasible, Subproblem 3.2 in the m th and Subproblem 3.2 in the $(m + 1)$ st iteration are feasible. Assume we fix \mathbf{z} , σ , ρ and \mathbf{p} in step 2 of the m th iteration. Given Subproblem 3.1 is feasible, the solution always exists and satisfies (27f). Fixing \mathbf{x} , σ , ρ and \mathbf{p} , the current \mathbf{z} and \mathbf{r} satisfy all the constraints in Subproblem 3.2 of the m th iteration. Therefore, Subproblem 3.2 of the m th iteration is feasible and the solution satisfies (28e). From (22), (24), (25) and the definition of $\mathbf{g}(\cdot)$, the updated \mathbf{y} and \mathbf{x} in step 2, \mathbf{z} and \mathbf{r} in step 3 result in $\mathbf{g}(\sigma) \leq \sigma$. Then, at the beginning of step 3, we have $\sigma^{(0)} = \hat{\sigma}$. Since $\mathbf{g}(\sigma^{(0)}) \leq \sigma^{(0)}$, Algorithm 5 in step 4 of the m th iteration converges according to Lemma 6, and the updated σ is smaller than $\sigma^{(0)}$ and satisfies (22) and (24). If the $(m + 1)$ st iteration is conduct, the current \mathbf{x} , \mathbf{y} , \mathbf{z} , \mathbf{r} , ρ , σ and \mathbf{p} satisfy (27f) in Subproblem 3.1. Thus, it is straightforward that Subproblem 3.1 of the $(m + 1)$ st iteration is feasible.

Next, from the preceding analysis, both steps 2, 3, and 4 results in non-increasing ρ which is lower-bounded by zero. Hence, Algorithm 5 converges.

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